

NEAR-EQUILIBRIUM TRANSPORT

Fundamentals and Applications

Lessons from Nanoscience: A Lecture Note Series

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(*Purdue University, USA*)

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by Supriyo Datta

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by Mark Lundstrom and Changwook Jeong

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A Lecture Note Series

Vol. 2

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Mark Lundstrom
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Fundamentals and Applications

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To

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and
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Preface

Engineers and scientists working on electronic materials and devices need a working knowledge of “near-equilibrium” (also called “linear” or “low-field”) transport. By “working knowledge” we mean understanding how to use theory in practice. Measurements of resistivity, conductivity, mobility, thermoelectric parameters as well as Hall effect measurements are commonly used to characterize electronic materials. Thermoelectric effects are the basis for important devices, and devices like transistors, which operate far from equilibrium, invariably contain low-field regions (e.g. the source and drain) that can limit device performance. These lectures are an introduction to near-equilibrium carrier transport using a novel, bottom up approach as developed by my colleague, Supriyo Datta and presented in Vol. 1 of this series [1]. Although written by two electrical engineers, it is our hope that these lectures are also accessible to students in physics, materials science, chemistry and other fields. Only a very basic understanding of solid-state physics, semiconductors, and electronic devices is assumed. Our notation follows standard practice in electrical engineering. For example, the symbol, “ q ”, is used to denote the magnitude of the charge on an electron and the term, Fermi level (E_F), is used for the chemical potential in the contacts.

The topic of near-equilibrium transport is easy to either over-simplify or to encumber by mathematical complexity that obscures the underlying physics. For example, ballistic transport is usually treated differently than diffusive transport, and this separation obscures the underlying unity and simplicity of the field. These lectures provide a different perspective on traditional concepts in electron transport in semiconductors and metals as well as a unified way to handle macroscale, microscale, and nanoscale devices. A short introduction to the Boltzmann Transport Equation (BTE), which

is commonly used to describe near-equilibrium transport, is also included and related to the approach used here. Throughout the lectures, concepts are illustrated with examples. For the most part, electron transport with a simple, parabolic energy band structure is assumed, but the approach is much more general. A short chapter shows, for example, how the same approach can be applied to the transport of heat by phonons, and to illustrate how the theory is applied to new problems. The lectures conclude with a case study – near-equilibrium transport in graphene.

It should, of course, be understood that this short set of lectures is only a starting point. The lectures seek to convey the essence of the subject and prepare students to learn. The additional topics needed to address specific research, development, and engineering problems on their own. Online versions of these lectures are available, along with an extensive set of additional resources for self-learners [2]. In the spirit of the *Lessons from Nanoscience* Lecture Note Series, these notes are presented in a still-evolving form, but we hope that readers find them a useful introduction to a topic in electronic materials and devices that continues to be relevant and interesting at the nanoscale.

*Mark Lundstrom
Changwook Jeong
Purdue University
June 18, 2012*

- [1] Supriyo Datta, *Lessons from Nanoelectronics: A new approach to transport theory*, Vol.1 in *Lessons from Nanoscience: A Lecture Notes Series*, World Scientific Publishing Company, Singapore, 2011.
- [2] M. Lundstrom, S. Datta, and M.A. Alam, “Lessons from Nanoscience: A Lecture Note Series”, <http://nanohub.org/topics/LessonsfromNanoscience>, 2011.

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